# Study of Constructivism and Epistemology in semiotic artificial intelligence

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## 1. Objectives:

The proposal refers to considerations of semiotic AI by Topos and Universal logic as a part of research on contribution expected from artificial intelligence AI by basic mathematics such as constructivism and epistemology. Artificial intelligence AI is classified into two types, non-symbolic AI and semiotic AI. The former include deep learning using multiple neural networks, and have been industrially successful. Mathematical logic research such as category theory universal logic by Topos contributes to the semiotic AI that does not rely on statistical data. This proposal deepens the perspective of Topos's logical substance, *hyperdoctrine*, and the category theory universal logic that makes it relative to the monad, and considers the role of duality in semantics. Specifically, we show that category theory logic fuses proof theory and model theory. Furthermore, the human language acquisition function is regarded as a mathematical object, and it aims to contribute to machine language acquisition and the integrations of semiotic AI and non-symbolic AI.

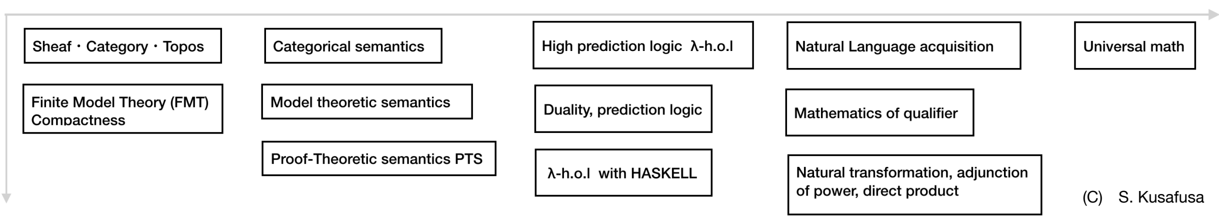
(Common ground of the research)

Once AI research started with logical semiotic, but decision making by mass data analysis was first put into practical use. This analysis is a complex process that requires large and diverse data sets. In the 21st century, machine learning and deep learning based on discrete structure processing have been put to practical use and are up to the present. On the other hand, in the field of pure mathematics, researches from sheaf, cohomology to category theory in the latter half of the twentieth century had been focused not only representation theory and logic, but also on linguistic semantics, λ-calculus, quantum mechanics, and systems. As of the 21st century, category theory that replaces set theory is changing the concept of the limit. Looking at patterns such as universality, duality, natural transformation, adjunction, recursion, fixed point, symmetry, and automorphism in mathematics construction by category theory also suggests that new theorems and breakthroughs may be introduced. Furthermore, the concept of symmetry as invariant is regarded as the essence of acquiring language functions, and it is assumed that pattern acquisition is the ultimate operation. Topos, like sheaf, is an extension of the modern concept of set, and Topos is integrating new mathematics. Topos is a category theory of intuitionistic logic (set theory), which embodies intuitionistic higher-order prediction logic. It is necessary to determine whether Topos that are relativized to logic can provide universal logic as a unified categorical perspective on various logics. It is hoped that these latest studies in mathematical logic will contribute essentially to semiotic AI.

## 2. Milestone chart:

Research design and methods consist of the following tasks. Review recent Topos studies. With category theory and philosophical considerations, conduct to find new hypotheses through thinking experiments on the following issues. Evaluate those hypotheses and verify algorithms by programming.

1) Published "Math is a language, from Sheaf, Category through to Topos"
2) Model theory and compactness, finite model theory (FMT)
3) Category theoretic semantics, Higher order prediction logic (λ-h.o.l), and Heyting algebra
4) Duality, semantics of prediction logic and set theory
   λ-h.o.l by Haskell programming, proof by theorems by Coq/SSReflect/MathCompx
5) Linguistics, generative grammar, minimal programs, and quantifier mathematics



**3. Implications and contribution to the math philosophy:**

(1) Published the math book "Math is a language" indicating Sheaf, Category and Topos

Mathematics is a language and proving a subject. In modern mathematics, the process of proving the theorem by a computer is based on a new relationship of formalization of mathematics. Understand that category theory, which replaces set theory, is deeply related to computational science and contributes to logic and linguistic semantics. Take a look at the tide from sheaf, cohomology to category theory, furthermore to Topos.

2) Model theory and compactness, finite model theory (FMT)

Survey the latest trends in model theory, which views first-order predicate logic as a finite group, closely related to model of computation, discrete mathematics and computational complexity theory. Recently, important results have been found for mathematical foundation and mathematical logic. Model theory studies aspects of axiom models, and studies theories by using models. Modern mathematical logic is a multifaceted subject, involving the strengths and limitations of formal proofs and algorithms, and the relationship between language and mathematical structure. It also deals with fundamental problems that arise in mathematics. Here, I'm trying to link the two logics of the latest model theory and set theory. Compactness is the most basic concept in model theory. Everything starts with this theorem. It is a set $T$ of closed logical expressions written in a language, and the Gödel's completeness theorem is equivalent that $T$ is consistently satisfiable and that $T$ has a model. I've proved this using the basic theorem of ultra products without using completeness theorem. In the age of discrete mathematics, locally finite Topos are the foundation for building discrete mathematics. In addition, I'm going to intimate that another model theory based on formalism, analogy, and reverse engineering methodology is useful.

3) Higher order prediction logic (λ-h.o.l), Heyting algebra

I claim that Topos (intuitionistic logic = constructivism by category theory), quantum linguistics, mathematical philosophy will contribute to the study of semiotic AI. First, we formulate the theorem of the construction pattern of mathematical logic. From the source of mathematical logic, I have shown the Kleene recursion theorem and the Rogers fixed-point theorem, and the relativity and automorphic forms. There are various types of logic, such as classical logic, intuitionistic logic, quantum logic, geometric logic, and fuzzy logic. Which of these is absolute logic is neither. I"m going to show that each system in logic only captures one aspect of the logical structure inherent in human cognitive mechanisms that cannot be captured by logical formalization (dwarfization).

Intuitionistic logic evolved in the logic of computer science, starting with the study of mathematical foundation. The pursuit of proof dynamism is constructivism in the computational science sense. The starting point was intuitionistic logic, along with a variety of tools devised (structural proof theory, feasibility interpretation, functional interpretation, Curry-Howard isomorphism, classical Intuitionism of logic Translation to logic etc.). Furthermore, referring to the proof theory of Heyting algebra, I assert the idea that proof is a program. I also show that the representational semantics of λ-calculus are almost synonymous with the study of Cartesian closed category.

4) Category theoretic semantics of duality, prediction logic and set theory

Stone duality is itself a category theoretic semantics. Broad duality, including the adjunction of various propositional logics, gives the corresponding predication logic or category theory model of set theory. Such a stone model can be applied to proof of consistency and proof of independence. When thinking about the duality of propositional logic, I have already mentioned the domain of prediction logic and set theory. For example, a functor taking an open set system that leads to stone duality between the category of the topological space and the category of the frame gives a *T-hyperdoctrine* corresponding to geometric logic. In addition, duality theory goes beyond intuitionistic logic and set theory to systematically provide duality models for various systems. By using category theory, it is possible to construct universal logic that has both high generality and conceptual clarity. In the circumstance a viewpoint of algebraic logic called propositional logic as a *monad,* and a viewpoint of category theoretic logic such as quantifier are alive. There, the binary opposition behind stone duality, syntactic and (modelistic) semantics, is progressively resolved, and the logic itself is independent of a particular form of representation, such as a proof system or Tarski model. Draw out the concept of The semantics show that the typed λ-calculus leads to a theorem that determines the functor that preserves the structure of the Cartesian closed category.

5) Linguistics, generative grammar, minimal programs, and quantifier mathematics

In human language acquisition, the concept of symmetry as an invariant is considered essential. The natural phenomena that linguistics are concerned with are biological phenomena, not non-organisms, and apply the mathematics of the determiner extended to the latest general dimension to the generation grammar research with the Chomsky minimal program. The determiner is conservative. We human beings have existential qualifiers. On the other hand, when considering the mathematical limit, conservation can be explained naturally. Understand that pattern acquisition, or language acquisition, is an extreme operation. Logic is representative of discrete structures. As an analogy, we will consider whether the limit can be applied to machine learning, which is formalized as a transformation between discrete structures.

One must be careful to conclude that Topos is literally universal logic. Until now, mathematicians, logicians, and philosophers have been studying in their respective fields, but I would like to pursue a more interdisciplinary, mathematical philosophical study and open up a new field of study called universal mathematics.

**4 . Inspection, sample programming:**

λ-h.o.l by Haskell programming, proof by theorems by Coq/SSReflect/MathCompx.

In this project, I will create higher order prediction logics by Haskell and try to prove, mathematical

In order to verify the results of this study, a higher-order logic by Haskell and a theorem proof program by Coq / SSReflect / MathComp are produced. Expanding the scope of proof by computer and showing a case of fusion with machine learning is worthy of evaluation. In addition, the programming used for verification will be used for mathematics, mathematical logic, and programming education for children who will be the next generation.

5. Records of preliminary research for this proposal:

The following papers on computer science and mathematical logic have been registered in the public archive. URL is http://imetrics.co.jp/academy/Blog5.pdf

*COVID-19 Math modeling and logic April 8, 2020*

*The research proposal of semiotic artificial intelligence Feb 25, 2020.*
*Self-referencing and fixed point theorem March 13, 2020.*
*Matryoshka and recursion Reconsider Recursively 2020. 3. 11*
*Criticism of pure reason and understanding March 8, 2020*
*Symmetry as an invariant is the essence of language acquisition. Feb 14, 2020*
*Monad Feb 28, 2020*
*Stone Duality and Hilbert's Nullstellensatz Aug 15, 2019*
*Mathematical logic, by Shoenfield 1967 July 20, 2019*
*Kleene's recursion theorem and Rogers's fixed-point theorem July 23, 2019*
*Recursion-fixed point-and-automorphism .m4v Aug 2, 2019*
*Cartesian-Closed-Category Aug 3, 2019*
*Direct sum, direct product April 17, 2019*
*Finite Model Theory March 20, 2019*
*Two Model Theories. March 18, 2019*
*Duality Feb 15, 2019*
*Fixed point theorem & recursive programming Jan 11, 2018*
*Semantics of programming Jan 9, 2018*

Feb 25, 2020
*S. Kusafusa*