Grothendiec prime number 57 and aliquot series

57 has an aliquot sum of 23 and is the first composite member of the 23-aliquot tree. Although 57 is not prime, it is jokingly known as the "Grothendieck prime" after a story in which mathematician Alexander Grothendieck supposedly gave it as an example of a particular prime number.

What is aliquot series?

The aliquot sequence starting with a positive integer k can be defined formally in terms of the sum-of-divisors function σ_1 or the aliquot sum function s in the following way:[1]

$s_0 = k$
$s_n = s(s_{n-1}) = \sigma_1(s_{n-1}) - s_{n-1}$ if $s_{n-1} > 0$,
and s(0) is undefined.
For example, the aliquot sequence of 10 is 10, 8, 7, 1, 0 because:
$\sigma_1(10) - 10 = 5 + 2 + 1 = 8$,
$\sigma_1(8) - 8 = 4 + 2 + 1 = 7$,
$\sigma_1(7) - 7 = 1$,
$\sigma_1(1) - 1 = 0$.

Many aliquot sequences terminate at zero; all such sequences necessarily end with a prime number followed by 1 (since the only proper divisor of a prime is 1), followed by 0 (since 1 has no proper divisors). See (sequence A080907 in the OEIS) for a list of such numbers up to 75.
There are a variety of ways in which an aliquot sequence might not terminate:
Numbers whose aliquot sequence terminates in a 1.

1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75

Many articles mentioned that Grothendieck took by mistake 57 as a prime number or was in error in choosing congruent numbers. those were incomplete.I am convinced that he wanted to say that it was 23 prime that appears at the beginning of the 57 aliquot sequence. The math enigma as much as he must have merely wanted to mention the interestingness of number theory that a pruned prime of a sequence appears rather than simply taking prime numbers as an example.

Someone's said, there are two kinds of mathematicians in the world. One is a frog and the other is a bird. Grothendiec can be a bird definitely. Because He was working on mathematics interdisciplinary. Algebraic geometry was rewritten by him. Arithmetic Geometry (Diophantine Geometry) was made by him as well..
He really worked on mathematics bird's-eye view.

Check aliquot series of 57 by using C++ programming

```
#include <iostream>
#include <math.h>
// Aliquot Sequence of Grothendiek's prime number
#include <bits/stdc++.h>
using namespace std;
int getSum(int n)
{
    int sum = 0;
    for (int i=1; i<=sqrt(n); i++)
    {
        if (n%i==0)
        {
            if (n/i == i)
            sum = sum + i;
            else
            {
                sum = sum + i;
                sum = sum + (n/i);
            }
        }
    }
    return sum - n;
}
void printAliquot(int n)
{
    printf("%d ", n);
    unordered_set<int> s;
    s.insert(n);
    int next = 0;
    while (n > 0)
    {
        n = getSum(n);
        if (s.find(n) != s.end())
        {
            cout << "\nRepeats with " << n;
            break;
        }
        cout << n << " ";
        s.insert(n);
    }
}

int main()
```

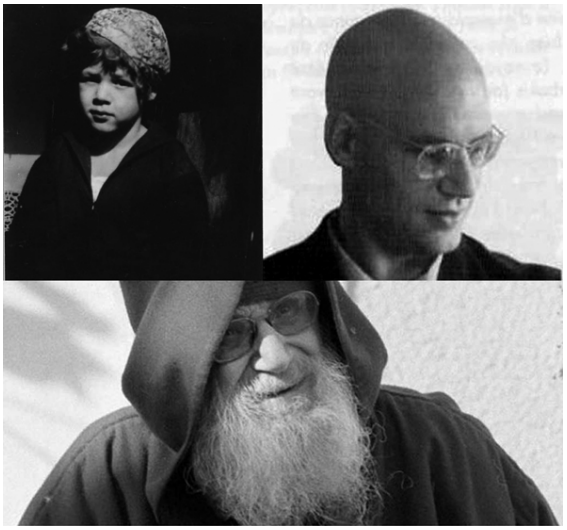
```
{
    printAliquot(57);
    return 0;
}
```

Calculation result:

```
57 23 1 0
```

Math Enigma, Alexander Grothendieck 28 March 1928 – 13 November 2014

Alexander Grothendiec was a German-born French mathematician who became the leading figure in the creation of modern algebraic geometry. His research extended the scope of the field and added elements of commutative algebra, homological algebra, sheaf theory and category theory to its foundations, while his so-called "relative" perspective led to revolutionary advances in many areas of pure mathematics. He is considered by many to be the greatest mathematician of the 20th century.



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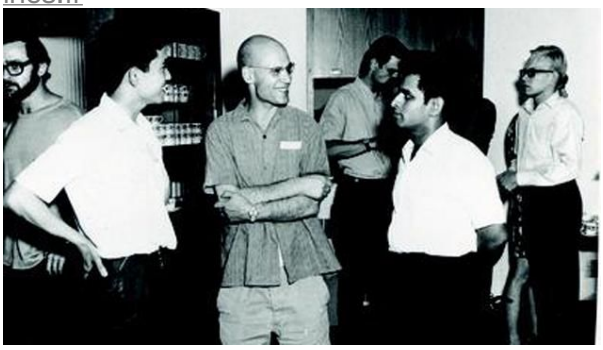
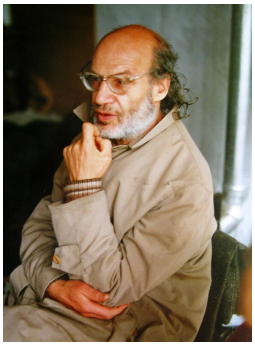


at 42 year-old, Montreal, 1970



Math Enigma, Dies at 86 year-old- The New York Times
Alexander Grothendieck in 1988. Credit Erika If

Сейджиро Кусафуса.

Links to relevance:

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