Matthew Frank の漸化式　recurrence relation

$$a(n)=a(n-1)+gcd(n,a(n-1))$$

ここで、gcd()は、最大公約数

初項 a(1) = 7

```
#include<stdio.h>
#include<math.h>
#include <stdlib.h>
#define TRIAL 100
int gcd(int x, int y)  {
  if(y == 0) return x;
  else return gcd(y, x%y);
}
int main() {
  int an,an_1,i;
  an_1=7;
  printf("a(1) = %d\n",an_1);
  for(i=2;i<TRIAL;i++){
    an=an_1 + gcd(i,an_1);
    printf("a(%d) = %d + gcd(%d,%d) = %d + %d = %d\n",
        i,an_1,i,an_1,an_1,gcd(i,an_1),an);
    an_1 = an;
  }
  return 0;
}
```

ideate.com での実行例

```
#include<stdio.h>
#include<math.h>
#include <stdlib.h>
#define TRIAL 100

int gcd(int x, int y)
{
  if(y == 0) return x;
  else return gcd(y, x%y);
}

```

```c
int main()
{
  int an,an_1,i;
  an_1=7;
  printf("a(1) = %d\n",an_1);
  for(i=2;i<TRIAL;i++){
    an=an_1 + gcd(i,an_1);
    printf("a(%d) = %d + gcd(%d,%d) = %d + %d = %d\n",
        i,an_1,i,an_1,an_1,gcd(i,an_1),an);
    an_1 = an;
  }
  return 0;
 }
```

計算結果:

```
a(1) = 7
a(2) = 7 + gcd(2,7) = 7 + 1 = 8
a(3) = 8 + gcd(3,8) = 8 + 1 = 9
a(4) = 9 + gcd(4,9) = 9 + 1 = 10
a(5) = 10 + gcd(5,10) = 10 + 5 = 15
a(6) = 15 + gcd(6,15) = 15 + 3 = 18
a(7) = 18 + gcd(7,18) = 18 + 1 = 19
a(8) = 19 + gcd(8,19) = 19 + 1 = 20
a(9) = 20 + gcd(9,20) = 20 + 1 = 21
a(10) = 21 + gcd(10,21) = 21 + 1 = 22
a(11) = 22 + gcd(11,22) = 22 + 11 = 33
a(12) = 33 + gcd(12,33) = 33 + 3 = 36
a(13) = 36 + gcd(13,36) = 36 + 1 = 37
a(14) = 37 + gcd(14,37) = 37 + 1 = 38
a(15) = 38 + gcd(15,38) = 38 + 1 = 39
a(16) = 39 + gcd(16,39) = 39 + 1 = 40
a(17) = 40 + gcd(17,40) = 40 + 1 = 41
a(18) = 41 + gcd(18,41) = 41 + 1 = 42
a(19) = 42 + gcd(19,42) = 42 + 1 = 43
a(20) = 43 + gcd(20,43) = 43 + 1 = 44
a(21) = 44 + gcd(21,44) = 44 + 1 = 45
a(22) = 45 + gcd(22,45) = 45 + 1 = 46
a(23) = 46 + gcd(23,46) = 46 + 23 = 69
a(24) = 69 + gcd(24,69) = 69 + 3 = 72
a(25) = 72 + gcd(25,72) = 72 + 1 = 73
a(26) = 73 + gcd(26,73) = 73 + 1 = 74
a(27) = 74 + gcd(27,74) = 74 + 1 = 75
a(28) = 75 + gcd(28,75) = 75 + 1 = 76
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a(29) = 76 + gcd(29,76) = 76 + 1 = 77
a(30) = 77 + gcd(30,77) = 77 + 1 = 78
a(31) = 78 + gcd(31,78) = 78 + 1 = 79
a(32) = 79 + gcd(32,79) = 79 + 1 = 80
a(33) = 80 + gcd(33,80) = 80 + 1 = 81
a(34) = 81 + gcd(34,81) = 81 + 1 = 82
a(35) = 82 + gcd(35,82) = 82 + 1 = 83
a(36) = 83 + gcd(36,83) = 83 + 1 = 84
a(37) = 84 + gcd(37,84) = 84 + 1 = 85
a(38) = 85 + gcd(38,85) = 85 + 1 = 86
a(39) = 86 + gcd(39,86) = 86 + 1 = 87
a(40) = 87 + gcd(40,87) = 87 + 1 = 88
a(41) = 88 + gcd(41,88) = 88 + 1 = 89
a(42) = 89 + gcd(42,89) = 89 + 1 = 90
a(43) = 90 + gcd(43,90) = 90 + 1 = 91
a(44) = 91 + gcd(44,91) = 91 + 1 = 92
a(45) = 92 + gcd(45,92) = 92 + 1 = 93
a(46) = 93 + gcd(46,93) = 93 + 1 = 94
a(47) = 94 + gcd(47,94) = 94 + 47 = 141
a(48) = 141 + gcd(48,141) = 141 + 3 = 144
a(49) = 144 + gcd(49,144) = 144 + 1 = 145
a(50) = 145 + gcd(50,145) = 145 + 5 = 150
a(51) = 150 + gcd(51,150) = 150 + 3 = 153
a(52) = 153 + gcd(52,153) = 153 + 1 = 154
a(53) = 154 + gcd(53,154) = 154 + 1 = 155
a(54) = 155 + gcd(54,155) = 155 + 1 = 156
a(55) = 156 + gcd(55,156) = 156 + 1 = 157
a(56) = 157 + gcd(56,157) = 157 + 1 = 158
a(57) = 158 + gcd(57,158) = 158 + 1 = 159
a(58) = 159 + gcd(58,159) = 159 + 1 = 160
a(59) = 160 + gcd(59,160) = 160 + 1 = 161
a(60) = 161 + gcd(60,161) = 161 + 1 = 162
a(61) = 162 + gcd(61,162) = 162 + 1 = 163
a(62) = 163 + gcd(62,163) = 163 + 1 = 164
a(63) = 164 + gcd(63,164) = 164 + 1 = 165
a(64) = 165 + gcd(64,165) = 165 + 1 = 166
a(65) = 166 + gcd(65,166) = 166 + 1 = 167
a(66) = 167 + gcd(66,167) = 167 + 1 = 168
a(67) = 168 + gcd(67,168) = 168 + 1 = 169
a(68) = 169 + gcd(68,169) = 169 + 1 = 170
a(69) = 170 + gcd(69,170) = 170 + 1 = 171
a(70) = 171 + gcd(70,171) = 171 + 1 = 172
a(71) = 172 + gcd(71,172) = 172 + 1 = 173
a(72) = 173 + gcd(72,173) = 173 + 1 = 174
a(73) = 174 + gcd(73,174) = 174 + 1 = 175
a(74) = 175 + gcd(74,175) = 175 + 1 = 176
a(75) = 176 + gcd(75,176) = 176 + 1 = 177
a(76) = 177 + gcd(76,177) = 177 + 1 = 178
a(77) = 178 + gcd(77,178) = 178 + 1 = 179

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a(78) = 179 + gcd(78,179) = 179 + 1 = 180
a(79) = 180 + gcd(79,180) = 180 + 1 = 181
a(80) = 181 + gcd(80,181) = 181 + 1 = 182
a(81) = 182 + gcd(81,182) = 182 + 1 = 183
a(82) = 183 + gcd(82,183) = 183 + 1 = 184
a(83) = 184 + gcd(83,184) = 184 + 1 = 185
a(84) = 185 + gcd(84,185) = 185 + 1 = 186
a(85) = 186 + gcd(85,186) = 186 + 1 = 187
a(86) = 187 + gcd(86,187) = 187 + 1 = 188
a(87) = 188 + gcd(87,188) = 188 + 1 = 189
a(88) = 189 + gcd(88,189) = 189 + 1 = 190
a(89) = 190 + gcd(89,190) = 190 + 1 = 191
a(90) = 191 + gcd(90,191) = 191 + 1 = 192
a(91) = 192 + gcd(91,192) = 192 + 1 = 193
a(92) = 193 + gcd(92,193) = 193 + 1 = 194
a(93) = 194 + gcd(93,194) = 194 + 1 = 195
a(94) = 195 + gcd(94,195) = 195 + 1 = 196
a(95) = 196 + gcd(95,196) = 196 + 1 = 197
a(96) = 197 + gcd(96,197) = 197 + 1 = 198
a(97) = 198 + gcd(97,198) = 198 + 1 = 199
a(98) = 199 + gcd(98,199) = 199 + 1 = 200
a(99) = 200 + gcd(99,200) = 200 + 1 = 201
```

漸化式2

```
#include<stdio.h>
#include<math.h>
#include <stdlib.h>
#define TRIAL 700
int gcd(int x, int y)  {
  if(y == 0) return x;
  else return gcd(y, x%y);
}
int main() {
  int an,an_1,i;
  int p;
  an_1=7;
  printf("a(1) = %d\n",an_1);
  for(i=2;i<TRIAL;i++){
    an=an_1 + gcd(i,an_1);
    p = gcd(i,an_1);
    printf("%d ", p);
    // printf("a(%d) = %d + gcd(%d,%d) = %d + %d = %d\n",
    //    i,an_1,i,an_1,an_1,gcd(i,an_1),an);
    an_1 = an;
  }
  return 0;
}
```

計算結果;　1と素数しか現れない!

a(1) = 7

1 1 1 5 3 1 1 1 1 11 3 1 1 1 1 1 1 1 1 1 1 23 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 47 3 1 5
3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
101 3 1 1 7 1 1 1 1 11 3 1 1 1 1 1 13 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 233 3 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1