Completeness

Japanese

A mathematical joke

In addition to the compactness mentioned earlier, discuss about the completeness which is the basic concept of mathematics. Perfect is a term similar to complete, which is used for general conversation. First, let's introduce a math joke.

"I'm a nobody, nobody is perfect, and therefore I'm perfect."

Since this syntax is "A = B", "B = C", and therefore "A = C", it means "I = perfect" because "I = a nobody", "nobody = perfect".

Nobody is used not only as a negative word meaning that there is no one, but also a meaningless person. In that case, a is attached to the definite article. Nobody is perfect means that there are no perfectionists. Although it is a joke using syllogism, logically it is not none. Most jokes use different syntax and semantic differences. In other words, the syllogality guarantees "logical correctness", but it does not guarantee the correctness of the content being stated. In addition, also refer to this joke.

"Business conventions are important because they demonstrate how many people a company can operate without."

This joke says in semantics. It is a mockery on how the meeting at the company is wasting time. People who do not know the office work may not understand the joke.

Operation to compensate for deficiency

Anyway, the opposite word of complete is incomplete, The complete antonym is incomplete. In general both are perfect / imperfect, complete / incomplete and indistinguishable. Complete is in a state where all the parts are complete, perfect is excellent. So complete < perfect.

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However, in common, it seems to be used in the abstract meaning "operation to compensate for deficiency".

Back to the main issue. In mathematics, completeness is defined differently in each mathematical field. However, in common, it seems to be used in the abstract meaning "operation to compensate for deficiency".

Completeness in logic

In mathematical logic and meta-logic, a formal system is called complete with respect to a particular property if every formula having the property can be derived using that system, i.e. is one of its theorems; otherwise the system is said to be incomplete. The term "complete" is also used without qualification, with differing meanings depending on the context, mostly referring to the property of semantical validity. Intuitively, a system is called complete in this particular sense, if it can derive every formula that is true. Kurt Gödel, Leon Henkin, and Emil Leon Post all published proofs of completeness.

Completeness in mathematical logic has two meanings.
1) Properties that can prove proofs that are true in formal logical systems
2) Properties that can prove affirmation or denial of arbitrary sentences that can be expressed in a formal logical system

According to the completeness theorem, Godel proved that any theory in first-order predicate logic is perfect in the former sense. The famous incompleteness theorem shows that a consistent and recursive theory including natural number theory can not be a complete system in the latter sense.

note:) For automated translation and AI research, we have to think from first-order predicate logic to higher-order predicate logic HOPL.

Completeness in abstract mathematics
Well, the completeness in mathematics is the same as the compactness introduced for the sake of simplifying the proof, and conditioning that everything is complete.

Completeness in mathematics can be thought with a specific meaning with respect to each object in various situations, and in each meaning you can think of operations called completeness for objects that are not complete.

% 1 The completeness of the real numbers, which implies that there are no "holes" in the real numbers.

% 2 Complete metric space, a metric space in which every Cauchy sequence converges.

% 3 Complete uniform space, a uniform space where every Cauchy net in converges or equivalently every Cauchy filter converges.

% 4 Complete measure, a measure space where every subset of every null set is measurable.

% 5 Completion (algebra), at an ideal.

% 6 Completeness (cryptography), a boolean function is said to be complete if the value of each output bit depends on all input bits. This is a desirable property to have in an encryption cipher, so that if one bit of the input plaintext is changed, every bit of the output (ciphertext) has an average of 50% probability of changing.

% 7 Completeness (statistics), a statistic that does not allow an unbiased estimator of zero.

% 8 Complete graph, an undirected graph in which every pair of vertices has exactly one edge connecting them.

% 9 Complete category, a category C where every diagram from a small category to C has a limit; it is cocomplete if every such functor has a colimit.

% 10 Completeness (order theory), a notion that generally refers to the existence of certain suprema or infima of some partially ordered set.

% 11 Complete variety, an algebraic variety that satisfies an analog of compactness.

% 12 Complete orthonormal basis, Incomplete orthogonal sets. Given a Hilbert space H and a set S of mutually orthogonal vectors in H, we can take the smallest closed linear subspace V of H containing S. Then S will be an orthogonal basis of V; which may of course be smaller than H itself, being an incomplete orthogonal set, or be H, when it is a complete orthogonal set.

% 13 Complete sequence, a type of integer sequence.

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Compact: コンパクト
Complete: 完備
Perfect: 完全
syllogism: 三段論法
First order prediction logic: 一階述語論理
High order prediction logic: 高階述語論理
Category theory: 圏論
complete space: 完備空間
complete metric space: 完備距離空間
cocomplete category: 余完備圏
functor: 関手
colimit: 余極限
suprema: 上限
infima:下限
complete variety: 完備代数多様体