

Thinking of Compactness

Space as used herein refers to a community formed by gathering a plurality of people. Due to the spread of the Internet since the beginning of the 21st century, a large-scale community is formed on the social networking unspecified number of people gather. But many of them have been established only by extremely limited users, or mostly users who exhausted the community, and it is known that they are not huge enough to be seen from the outside. As to community such as real finite and closed circumstances, Mathematician Heine Borel named the irony "compact space". Nikola Burubaki further advocated "a community where each user builds a mind barrel and takes a distance and will not hold a conclusion with anyone" as a stronger definition. But until this point, it seems that it cannot be said that it is an community anymore. However, note that Burubaki himself has developed personality schizophrenia and incoherent in words.

Heine Borel's theorem:

"Finite and closed space falls into a compact space, further promoting a vicious circle" When limiting the number of members of the community or suppressing new participants, gradual ranking among members has occurred and several are influential people It will reign as. General members are managed under the name of "Protect Order" by influential people, and while enjoying peace, they are restricted in their actions. It is completely covered by influential people. This covering is opaque, which becomes a factor to keep people outside.

In real analysis the Heine–Borel theorem, named after Eduard Heine and Émile Borel, states: For a subset S of Euclidean space R^n , the following two statements are equivalent:

- S is closed and bounded
- S is compact, that is, open covering of S has a finite sub-covering.

In the metric space, compactness of the subset and complete equality bounded are equivalent

Bolzano-Weierstrass's theorem:

"The prolonged debate can make it converge to the best conclusion if you thin out participants appropriately." Even in a hot and debated discussion, after all it is only a storm in the cup. The reason why the argument that should be calm down naturally even if it is left is prolonged is because the insidious people gather and gather. Such a senior is not only "a user who exhausts the community" but the theory advocated by the influential person of ruling party who is required to be removed promptly by the administrator.

Although the above is an ironic definition of the compact, they are not mistakes. The concept of compact is described below briefly.

The expression "compact" is used as "compact car", "this city is compact in everything", "compactly summarize luggage of travel", but what is compact? (X, d) is a metric space and $A \subset X$ is a subset, A is a compact set, arbitrarily gives the family of open sets $\{U_\lambda\}_{\lambda \in \Lambda}$ that is $A \subset \bigcup_{\lambda \in \Lambda} U_\lambda$ When, There is a finite number of $U_{\lambda_1}, U_{\lambda_2}, \dots, U_{\lambda_r}$ ($\lambda_1, \lambda_2, \dots, \lambda_r \in \Lambda, r$ is any natural number) in a part of the family of open sets, $A \subset \bigcup_{i=1}^r U_{\lambda_i}$ It is good when it comes to. It is a kind of manageability concept. Most definitions are not necessarily true unless they are compact..

In a certain local cities, for the time being, it is supposed to be able to cover the usual life with finite somethings of necessities for urban life. Let's call a compact space. Of course you cannot prepare everything indefinitely.

A lot of foreigners such as student and researcher stay in this town though, few English sign, people doesn't speak English or promising foreign languages. Internationality is an essential covering condition. It is a strange story that it is not compact in an optional choice. The reason for introducing the compact is that it does not complicate the object.

In the definition of compactness, there was a doubt whether how to pick up the family of open set $\{U_\lambda\}_{\lambda \in \Lambda}$ (open covering) which is $A \subset \bigcup_{\lambda \in \Lambda} U_\lambda$, but the answer is arbitrary. The definition of compact set is that it is possible to choose a finite partial open covering for any open covering.

Also such a definition as following is negative. Compact set ($A \Leftrightarrow \exists U_{\lambda_1}, \dots, U_{\lambda_n}$, s.t. U_{λ_i} is open set ($1 \leq i \leq n$) and $A \subset \bigcup_{i=1}^n U_{\lambda_i}$). Not to be open covering of A, but as to arbitral covering of A. Open interval $(0, 1) \subset \mathbb{R}$ is not compact.

That is because there is a certain open covering, and a finite partial open covering can not be taken. For example, $U_\lambda = (0, \lambda)$, $1/2 \leq \lambda < 1$ defines such an open covering. From there it is impossible to obtain a finite partial open covering. Where $\lambda < 1$. In this case $\Lambda = [1/2, 1)$ is the set of subscripts.

The concept of "metric space" is not used in the definition of "compact", but there is a question as to whether (X, d) metric space is necessary. This answer is OK if X is a topological space without metric space. In the definition of the compact, U_λ and U_{λ_1} are $\lambda \in \Lambda$ and $\lambda_1 \in \Lambda$. Both are members of open covering.

The answer to the question that what is the definition of open covering is, The open covering of $A \subset X$ is a collection of open sets of $\{U_\lambda\}_{\lambda \in \Lambda}$ (any cardinal number of Λ can be used), covering A, that is $A \subset \bigcup_{\lambda \in \Lambda} U_\lambda$. The compact is the ultimate concept obtained as a result of attracting the essence of \mathbb{R}^n 's bounded closed set.

A topological space is a sequentially compact means that an arbitrary point sequence in the space includes a partial sequence converged. In the general topological space, the point sequence compactness and compactness are different concepts, but in the distance space only these two values are equivalent.

In general topology, compactness is a property that generalizes the notion of a subset of Euclidean space being closed that is, containing all its limit points and bounded that is, having all its points lie within some fixed distance of each other. Examples include a closed interval, a rectangle, or a finite set of points. This notion is defined for more general topological spaces than Euclidean space in various ways. One such generalization is that a topological space is sequentially compact if every infinite sequence of points sampled from the space has an infinite subsequence that converges to some point of the space. The Bolzano–Weierstrass theorem states that a subset of Euclidean space is compact in this sequential sense if and only if it is closed and bounded. Thus, if one chooses an infinite number of points in the closed unit interval $[0, 1]$ some of those points will get arbitrarily close to some real number in that space. For instance, some of the numbers $1/2, 4/5, 1/3, 5/6, 1/4, 6/7, \dots$ accumulate to 0 (others accumulate to 1). The same set of points would not accumulate to any point of the open unit interval $(0, 1)$; so the open unit interval is not compact. Euclidean space itself is not compact since it is not bounded. In particular, the sequence of points $0, 1, 2, 3, \dots$ has no subsequence that converges to any given real number.

A topological space is sequentially compact means that an arbitrary point sequence in the space includes a partial sequence converged. In the general topological space, the point sequence compactness and compactness are different concepts, but in the distance space only these two values are equivalent. I think that this definition explains the topic of the ironic community that I mentioned first.

Finally, I introduce the definition of another compact., that is, a concept of compactness in first-order predicate logic. The compactness theorem states that a set of first-order sentences has a model if and only if every finite subset of it has a model. This theorem is an important tool in model theory, as it provides a useful method for constructing models of any set of sentences that is finitely consistent.

Reference:

Completeness <http://imetrics.co.jp/opinion2/Completeness.pdf>

完備性について、<http://imetrics.co.jp/opinion2/完備性について.pdf>